Modeling Vaccination for a Novel Disease

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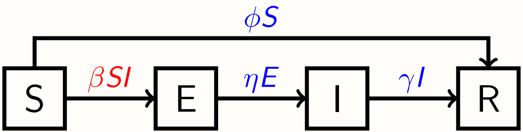
0. Standard Vaccination Model



$$
\begin{aligned}
S' &= -\beta SI - \phi S, \qquad & S(0) &= S_0;\\
E' &= \beta SI - \eta E, \qquad & E(0) &= E_0;\\
I' &= \eta E - \gamma I, \qquad & I(0) &= I_0.
\end{aligned}
$$

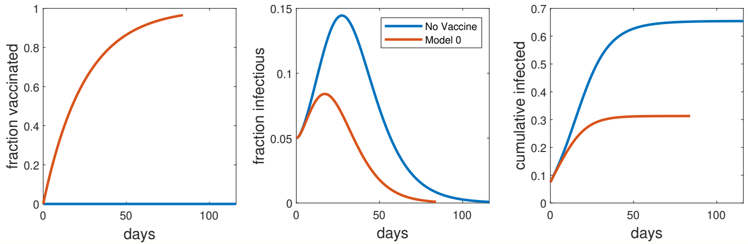
- Blue processes are instantaneous transitions.
  - Times are exponentially distributed.

Disclaimer

- The SEIR model is a little too simple for COVID-19.
- Nevertheless, we'll use parameters that roughly match COVID-19 in January 2021:
  - 5-day mean incubation
  - 10-day mean duration of infectivity
  - About 30% initial immunity from prior infection
  - Effective basic reproductive number for the delta variant (with masks and social distancing) roughly 2.5 to 4.0, depending on the community.
- The graphs for a COVID-19 model would be similar.

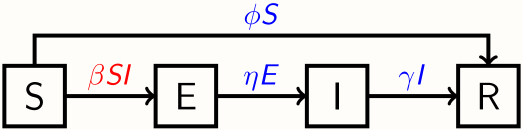
0. Vaccination Impact in Standard Model

- $\phi = 0.04$ — mean time for vaccination is 25 days.
- Effective reproductive number 4.0, corresponding to delta with limited mask use and social distancing.



- Vaccination looks really effective! But...

0. Standard Vaccination Model

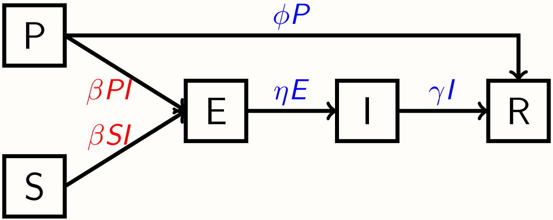


- What is wrong with this model?

  1. Nobody refuses the vaccine.
  2. Distribution is instantaneous.
  3. Supply is unlimited.
  4. Vaccine always confers immunity.

1. Vaccine Refusal

- Partition the susceptible class into a vaccine refuser group (S) and a prevaccination group (P).

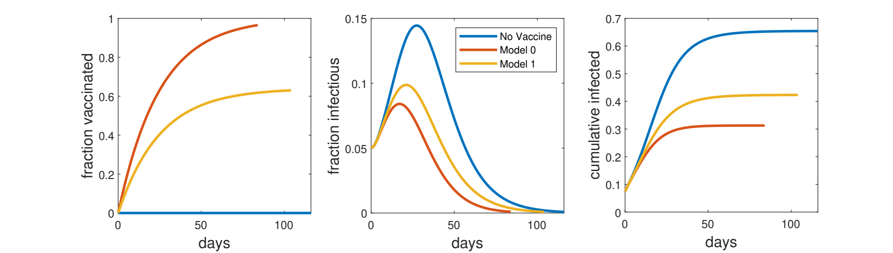


- Assume the time zero susceptible fraction is S_0 and that a fraction r of these are vaccine refusers.

$$S(0) = rS_0, \qquad P(0) = (1 - r)S_0.$$

1. Vaccine Refusal Impact

- Take $r = 0.36$ for the US average (near 0.1 in Portugal and Chile, and only because the vaccine is age-limited)

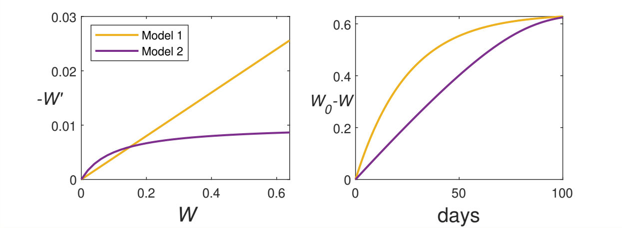


- Vaccine refusal makes a big difference, but....
- The vaccination rate should not depend on r until prevaccinated people are largely done — we need a better distribution model.

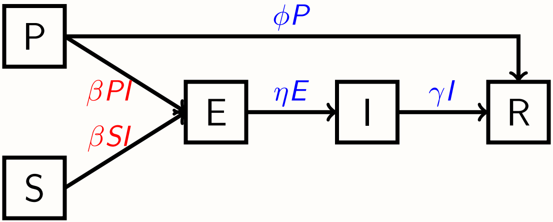
A Vaccination Model with Limited Distribution

- Let W be the fraction of people who want vaccination.
- Finding recipients for a vaccinator is like finding substrate for a biochemical enzyme (Michaelis-Menten kinetics).
  - K is the level of W for which the rate is half of the maximum.
  - V is a vaccination rate constant (1/time).

$$W' = -\frac{VKW}{K+W} \equiv -\phi W, \qquad W(0) = W_0 = 1 - r.$$



2. Limited Distribution Model

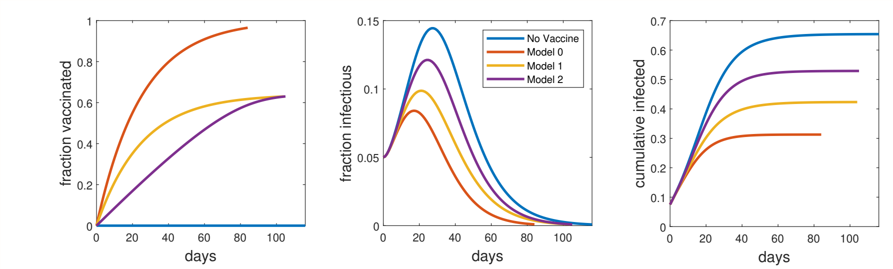


$$
\begin{aligned}
P' &= -\beta PI - \phi(W)P, & P(0) &= (1-r)S_0; \\
S' &= -\beta SI, & S(0) &= rS_0; \\
E' &= \beta(P+S)I - \eta E, & E(0) &= E_0; \\
I' &= \eta E - \gamma I, & I(0) &= I_0;
\end{aligned}
$$

$$
W' = -\phi(W)W, \qquad \phi(W) = \frac{VK}{K+W}, \qquad W(0) = 1 - r.
$$

2. Impact of Limited Distribution

- $V = 0.1, \quad K = 0.1$ — max vax rate is 1% per day

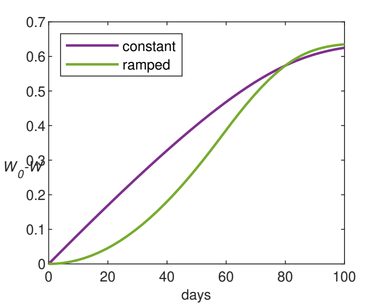


- Slow distribution significantly decreases vaccine impact.

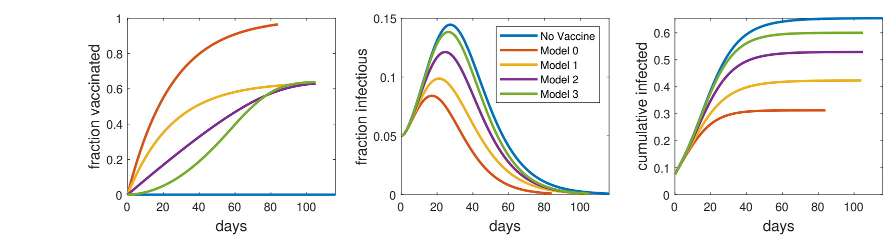
3. Limited Supply

- Vaccine production capacity increases over time.
  - The maximum vaccination rate should ramp up to a maximum.

$$V = \min\left(\frac{V_1 t}{t_1}, V_1\right).$$

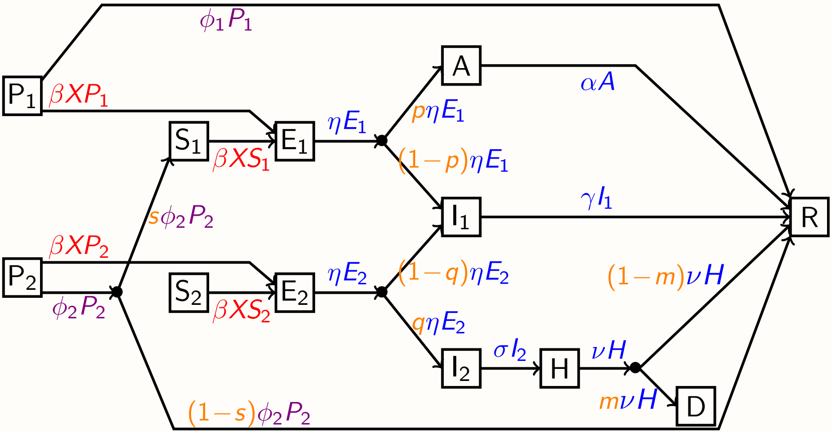


3. Impact of Limited Supply



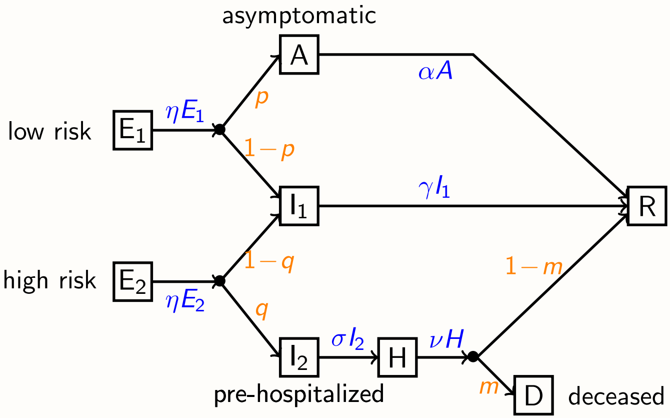
- If we want meaningful results, we need to use a realistic vaccination model.
- This scenario assumed a quick relaxing of mitigation strategies (think "Florida").
  - **Mitigation must be maintained during vaccination.**

The PSEAIHRD Model (COVID-19, January 2021)

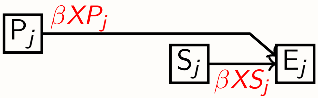


transitions, transmissions, vaccination, probabilities

The PSEAIHRD Model – Transitions



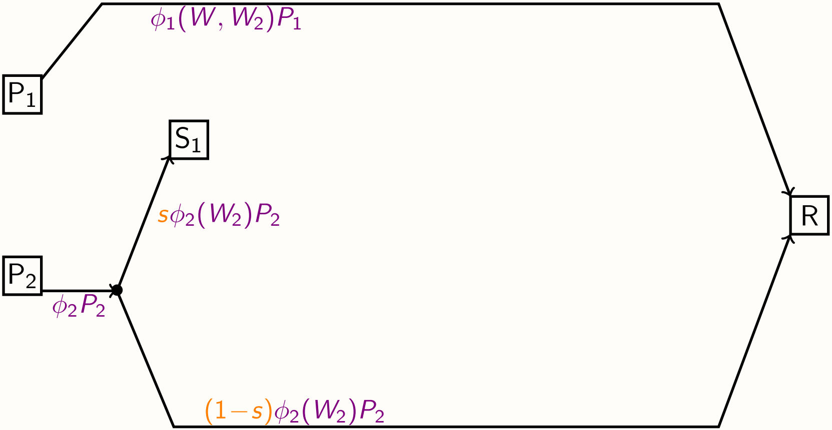
The PSEAIHRD Model – Transmissions



$$X = f_c(c_i I + c_a A) + \delta[(1 - c_i)I + f_a(1 - c_a)A].$$

- c_i and c_a are the fractions of confirmed cases for symptomatic and asymptomatic infectives.
- f_c, f_a are the infectivities of confirmed cases and asymptomatic cases, relative to an unconfirmed symptomatic infective.
- δ is a 'contact factor' that incorporates physical distancing and mask use for unconfirmed cases.

The PSEAIHRD Model – Vaccination



- The coefficients ϕ_i depend on the status of the overall (W) and high-risk (W_2) vaccination programs.

Two-Class Vaccination Model

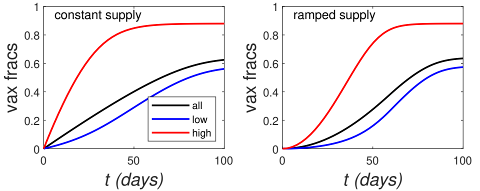
- Suppose a fraction h of the population is high-risk, with refusal probability $r_2 \leq r$.

$$W(0) = 1 - r, \quad W_2(0) = (1 - r_2)h.$$

- Assume the same model for high-risk as for the whole group.

$$W' = -\frac{VK}{K + W} \equiv -VG(W)W, \quad W_2' = -VG(W_2)W_2.$$

  - Then
    $$W_1' = \cdots = -VG(W)G(W_2)W_1, \quad W_1(0) = (1 - r_1)(1 - h).$$



PSEAIHRD Vaccination Details

- We need ϕ_j for
$$P_j' = -\phi_j P_j - \beta X P_j.$$
- We have
$$W_1' = -VG(W)G(W_2)W_1, \qquad W_2' = -VG(W_2)W_2,$$
- Therefore
$$\phi_1 = VG(W)G(W_2), \qquad \phi_2 = VG(W_2).$$
- We need only couple the W and W_2 equations to the PSEAIHRD system.

Resources

- See **https://www.math.unl.edu/SIR-modeling** for
  - Details on a classroom activity in disease modeling;
  - SIR and SEIR teaching modules using Excel and Matlab;
- See **https://www.math.unl.edu/covid-module** for COVID-19 teaching modules.
- Ledder and Homp, Using a COVID-19 model in various classroom settings to assess effects of interventions, PRIMUS 2021
  https://www.tandfonline.com/doi/full/10.1080/10511970.2020.1861143
- Ledder and Homp, Mathematical epidemiology, in Mathematics Research for the Beginning Student Volume 1: Accessible Research Projects for First- and Second-Year College and Community College Students before Calculus, ed. E.E. Goldwyn, A. Wootton, S. Ganzell. Birkhauser, in press

Resources

- Ledder, Mathematical Modeling for Epidemiology and Ecology, 2ed, Springer, in press
  1. Modeling in Biology
  2. Empirical Modeling
  3. Mechanistic Modeling
     - 3.1 Transition processes (*includes vaccination*)
     - 3.2 Interaction processes
     - 3.3 Compartment analysis: The SEIR epidemic model
     - 3.4 SEIR model analysis
     - 3.5 Two scenarios from the COVID-19 pandemic
     - 3.9 Adding demographics to make an endemic disease model
  4. Dynamics of Single Populations
  5. Discrete Linear Systems
  6. Nonlinear Dynamical Systems
- **Shoot me an email to receive updates or offer feedback!** gledder@unl.edu