Benefits of Mycorrhizal Associations Across a Spectrum of Environment Quality

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A Theoretical Question

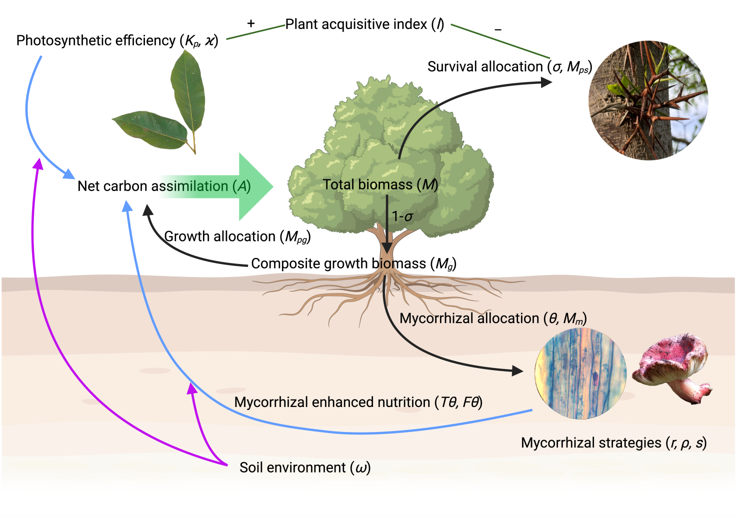
- Plant species occupy a spectrum of strategies:
  - **Acquisitive**: Use nearly all resources to increase growth/reproduction.
  - **Conservative**: Use a significant amount of resources to enhance survival.
- **Mycorrhizae** are root fungi that use carbohydrates from the plant and take up resources from the soil that plants can use.
- Research question: What effect do mycorrhizae have on the relative success of acquisitive and conservative plants?

Biological Setting

- Photosynthate is made in the leaves of plants using resources obtained from the roots, and can be allocated to:

  1. Structures that increase photosynthetic capacity;
     - Leaves, fine roots, and the transport system that connects different plant tissues
  2. Structures that help the plant's survival chances;
     - Water storage tissues, physical defenses against weathering, physical defenses against herbivory, and chemical defenses against herbivory
  3. Mycorrhizae, fungi on the roots that facilitate collection of nutrients.

Plant-Mycorrhiza System



Research Agenda

1. Build a model that predicts plant growth rate in terms of
   - Allocations to survival and mycorrhiza,
   - A small set of plant traits,
   - A small set of mycorrhizal traits,
   - The quality of the soil environment.

2. Use the model to predict when allocation to mycorrhiza is beneficial and the extent of that benefit.

3. Compare the value of mycorrhiza for plants covering a spectrum from acquisitive (higher investment in growth) to conservative (higher investment in survival).

Plant-Mycorrhiza System Components

Total biomass M consists of plant mass M_p, the latter subdivided into survival and growth compartments M_{ps} and M_{pg}, and mycorrhiza mass M_m.

1. **A fraction σ of photosynthate is allocated to survival**:
   - $M_{ps} = \sigma M$ is the biomass of survival components of the plant.
   - $M_g = (1 - \sigma)M$ **is the biomass allocated to growth.**

2. **A fraction θ of the growth biomass is allocated to mycorrhiza**:
   - $M_m = \theta M_g$ is the biomass of the mycorrhiza.
   - $M_{pg} = (1 - \theta)M_g$ is the biomass of the growth components of the plant.

Growth Model

The total system mass increases at the rate of photosynthate production.

$$\frac{dM}{dt} = \omega \cdot K \cdot C_0 M_{pg}^{2/3}.$$

- $C_0 M_{pg}^{2/3}$ is the **total leaf area** corresponding to the plant growth biomass.
- K is the **rate of photosynthesis per unit leaf area** in an ideal environment, **including value added by mycorrhiza**.
- ω represents the environment quality as a **multiplicative factor**.

Relative Growth Rate

We want the growth rate of the **plant**, not the whole system:

- We replace M and M_{pg} in the growth model with M_p:

$$\frac{dM_p}{dt} = \omega \, K_{max}(\kappa + \kappa_m)\, \gamma^{2/3}(1-\theta)^{2/3}(1-\gamma\theta)^{1/3} C_0 M_p^{2/3}.$$

  - κ is the intrinsic plant growth rate relative to K_{max},
  - κ_m is the value added by the mycorrhiza.
  - $\gamma = 1 - \sigma$ is the total growth allocation.

- With mycorrhiza, the relative growth rate at $\omega = 1$ is

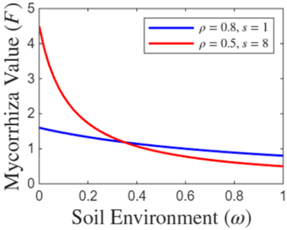
$$Q = (\kappa + \kappa_m)\, \gamma^{2/3}(1-\theta)^{2/3}(1-\gamma\theta)^{1/3}.$$

Mycorrhiza Value

Mycorrhiza are characterized in the model by two traits:

- ρ is the relative benefit per mycorrhiza mass in an ideal environment.
- s is the proportional increase in relative benefit as $\omega \to 0$.

$$\kappa_m = F(\omega;\rho,s)\cdot\theta, \quad F(\omega;\rho,s) = \left(\frac{1+s}{1+s\omega}\right)\rho.$$



Acquisivity Index

Relative growth rate:

$$Q = [\kappa + F(\omega;\rho,s)\theta]\,\gamma^{2/3}\,(1-\theta)^{2/3}\,(1-\gamma\theta)^{1/3}.$$

- Plants that have fast photosynthesis (large κ) generally invest less in survival (small σ).
  - We assume κ, σ are linear functions of an **acquisivity index** I:

$$0 \le \sigma = 0.4 - 0.4I \le 0.4, \quad 0 \le \kappa = 0.2 + 0.8I \le 1.$$

Now the growth rate depends only on

1. **Allocation parameter** θ
2. **Plant acquisivity** I
3. **Combined mycorrhiza/environment parameter** $F(\omega;\rho,s)$

Optimal Allocation

- **The optimal mycorrhiza allocation θ^* is determined by**
  - $I \in [0,1]$: **the plant acquisivity index**;
  - $F(\omega; \rho, s)$: **the relative mycorrhiza value in environment** ω

- There are two cases:

  1. **The mycorrhiza is not useful ($\theta^* = 0$) if**

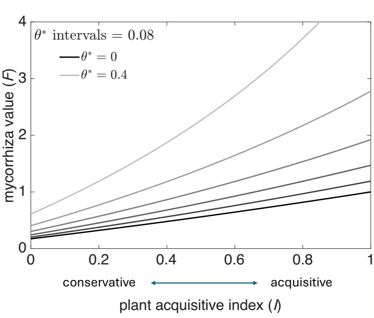
     $$F(\omega; \rho, s) \leq P(I) \left\{ \equiv \kappa \left(1 - \frac{\sigma}{3}\right) = \frac{(1+4I)(13+2I)}{75} \right\}.$$

     - $P(I) \leq 1$ is the plant's (relative) *inherent productive capacity*.

  2. **There exists a unique $\theta^* \in (0,1)$ that maximizes Q iff**

     $$F(\omega; \rho, s) > P(I).$$

Optimal Allocation

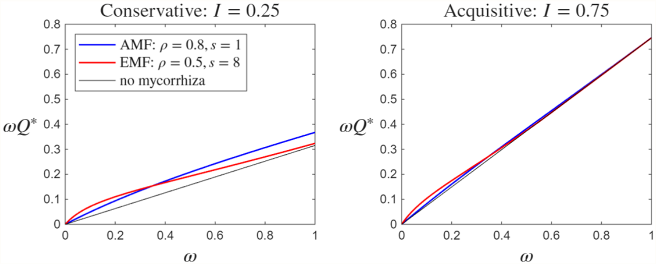


- The bottom curve is the inherent productive capacity $P(I)$.

1. **Conservative plants benefit more from mycorrhiza.**
   - The vertical spread is smaller at left than right.
2. **Increased mycorrhiza allocation has diminishing returns.**
   - The vertical spread is larger at top than bottom.

Growth Rates with Optimal Allocation to Mycorrhiza

[ω is environment quality, ωQ^* is growth rate with best allocation.]



3. **Conservative plants benefit from mycorrhiza in any environment.**
4. Acquisitive plants benefit only from **high-sensitivity** mycorrhiza (EMF) in **poor environments.**

Reference

Yang, Y. Ledder, G., and Russo, S.E. Benefits and costs of mycorrhizal associations vary along the acquisitive-conservative plant strategy spectrum. *Ecological Monographs*, in press.

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