A COVID-19 Model for January 2021

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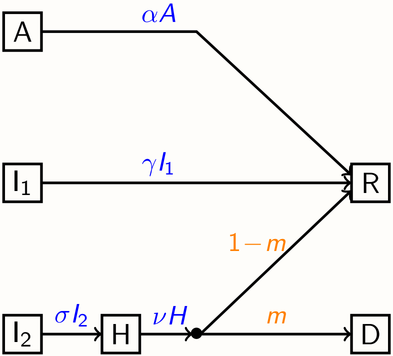
Key Differences from SEIR

- Some patients are asymptomatic, with lower infectivity and shorter infectious duration.
- We need to track hospitalizations.
- Public health policies can decrease infectivity, but in different ways for different classes.
- Vaccination is subject to supply limitations and administrative time delays.

Multiple Infectious Classes

- Asymptomatic (A)
  - Shorter duration, reduced infectivity, less likely to be tested
  - Always recovers
- Symptomatic (I_1)
  - Base duration and infectivity, more likely to be tested
  - Always recovers
- Pre-Hospitalized (I_2)
  - Base infectivity, more likely to be tested
  - Always becomes hospitalized
- Hospitalized (H)
  - Limited infectivity
  - Either dies (probability m) or recovers

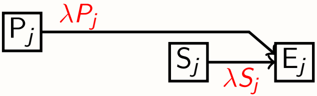
A PSEAIHRD Model – Multiple Infectious Classes



Infectivity Details

- We replace the factor I in the SEIR transmission rate formula βSI with an 'effective infective population' X. Without mitigation, $X = I + f_a A$, where $f_a < 1$.
- Testing
  - Isolation lowers infectivity from 1 (I) or f_a (A) to f_c
- Risk Reduction
  - Social distancing reduces contact rates of untested individuals (A and I)
  - Masking reduces infectivity of untested individuals (A and I)
  - Contribution of untested A and I individuals is reduced by a factor δ.

A PSEAIHRD Model – Transmissions



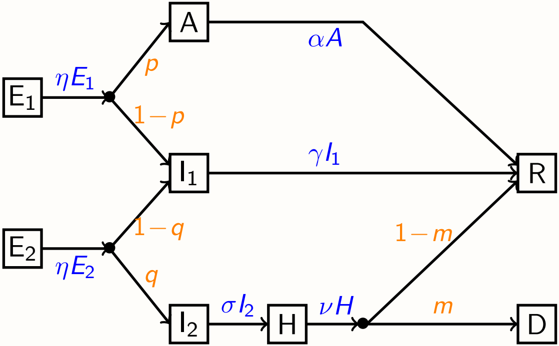
$$\lambda = \beta X, \quad X = f_c(c_i I + c_a A) + \delta[(1 - c_i)I + f_a(1 - c_a)A].$$

- c_i and c_a are the fractions of confirmed cases for symptomatic and asymptomatic infectives.
- f_c, f_a are the infectivities of confirmed cases and asymptomatic cases, relative to an unconfirmed symptomatic infective.
- δ is a 'contact factor' that incorporates social distancing and mask use for unconfirmed cases.

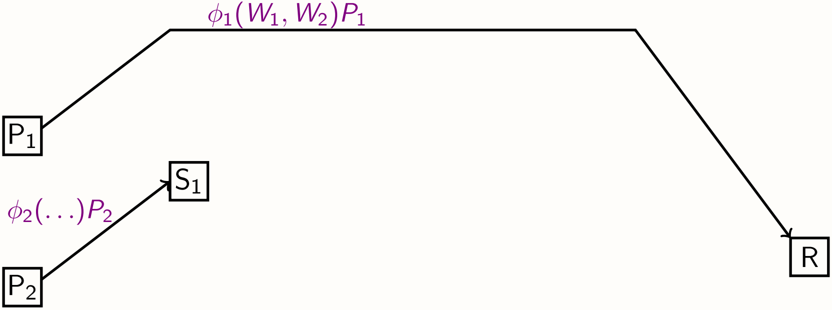
Multiple Risk Classes

- Low Risk (P_1, S_1, E_1)
  - Goes to asymptomatic A (probability p) or symptomatic I_1
  - Vaccination confers immunity
- High Risk (P_2, S_2, E_2)
  - Goes to prehospitalized I_2 (probability q) or symptomatic I_1
  - Vaccination decreases risk category
- We are neglecting other possibilities under the assumption that worse outcomes (vaccination doesn't do anything) roughly balance better outcomes (high risk becomes immune).

A PSEAIHRD Model – Transitions and Decisions

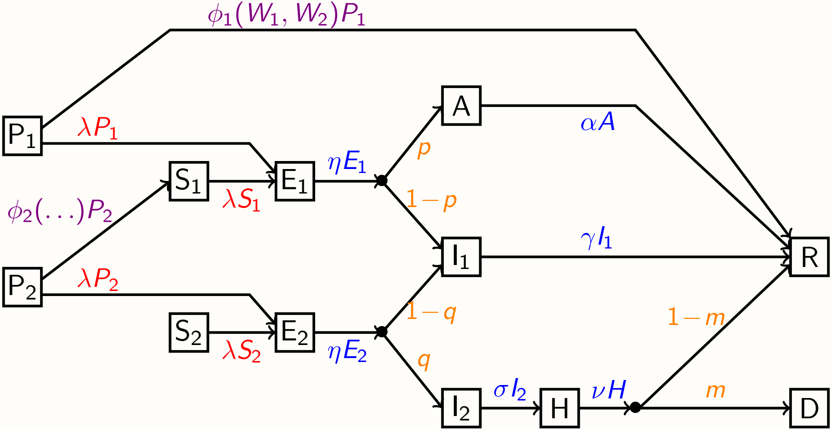


A PSEAIHRD Model – Vaccination



- $\phi_i(W_1, W_2)$ are rate 'constants' that depend on the status of the vaccination program.
  - W_1 and W_2 are the population fractions of low/high risk people waiting for vaccinated.

Full PSEAIHRD Model

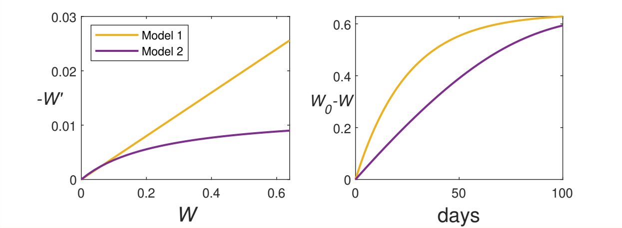


transitions, decision points, transmissions, vaccination

A Vaccination Model

- Let W be the fraction of people who want vaccination.
- Finding recipients for a vaccinator is like finding substrate for a biochemical enzyme (Michaelis-Menten* kinetics).
  - K is the level of W for which the rate is half of the maximum.
  - V is a vaccination rate constant (1/time).

$$W' = -\frac{VKW}{K+W} \equiv -\phi W, \qquad W(0) = W_0 = 1 - r.$$



Two-Class Vaccination Model

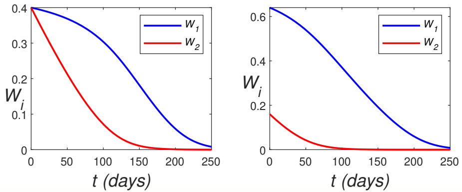
- τ is high-risk fraction, r_j is fraction that refuses vaccination.

$$W_2(0) = (1 - r_2)\tau, \quad W_1(0) = (1 - r_1)\tau.$$

- Assume the same model for high-risk as for the whole group.

$$W_2' = -\frac{V}{K + W_2} W_2 \equiv -v(W_2)W_2, \quad v(W_2) = \frac{V}{K + W_2}.$$

  - Then
    $$W_1' = W' - W_2' = \cdots = -k(W)v(W_2)W_1, \quad k(W) = \frac{K}{K + W}.$$



PSEAIHRD Vaccination Details

- We need ϕ_j for
$$P_j' = -\phi_j P_j - \lambda P_j.$$
- We have
$$W_1' = -k(W)v(W_2)W_1, \qquad W_2' = -v(W_2)W_2,$$
$$v(W_2) = \frac{V}{K + W_2}, \qquad k(W) = \frac{K}{K + W}.$$
- Therefore
$$\phi_2 = v(W_2), \qquad \phi_1 = k(W)v(W_2).$$
- We need only couple the W and W_2 equations to the PSEAIHRD system.

Pedagogical Resources

- See **https://www.math.unl.edu/SIR-modeling** for
  - Details on a classroom activity in disease modeling;
  - SIR and SEIR teaching modules using Excel and Matlab;
- See **https://www.math.unl.edu/covid-module** for COVID-19 teaching modules.
- Ledder and Homp, Using a COVID-19 model in various classroom settings to assess effects of interventions, PRIMUS 2021
  https://www.tandfonline.com/doi/full/10.1080/10511970.2020.1861143
- Ledder and Homp, Mathematical epidemiology, in Mathematics Research for the Beginning Student Volume 1: Accessible Research Projects for First- and Second-Year College and Community College Students before Calculus, ed. E.E. Goldwyn, A. Wootton, S. Ganzell. Birkhauser, in press

Resource for Modeling and Analysis

- Ledder, Mathematical Modeling for Epidemiology and Ecology, 2ed, Springer, in press
  1. Modeling in Biology
  2. Empirical Modeling
  3. Mechanistic Modeling
     - 3.1 Transition processes
     - 3.2 Interaction processes
     - 3.3 Compartment analysis: The SEIR epidemic model
     - 3.4 SEIR model analysis
     - 3.5 Two scenarios from the COVID-19 pandemic
     - 3.9 Adding demographics to make an endemic disease model
  4. Dynamics of Single Populations
  5. Discrete Linear Systems
  6. Nonlinear Dynamical Systems
     - Nullcline analysis and linearized stability analysis
- **Shoot me an email to receive updates or offer feedback!**
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