# Parameterizing Epidemic Models

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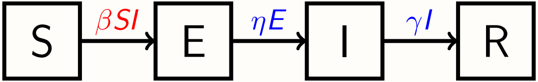
# SEIR epidemic model




$$S' = -\beta S I$$

$$E' = \beta S I - \eta E$$

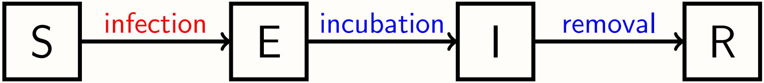
$$I' = \eta E - \gamma I$$

$$R' = \gamma I$$

- Let $N = S + E + I + R$. Then $N' = 0$, so $N$ is constant.
  - The $R$ equation is not needed because $R = N - S - E - I$.

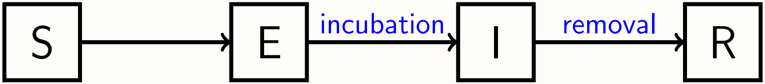
# Processes

- Processes are either **transmissions** or **transitions**.



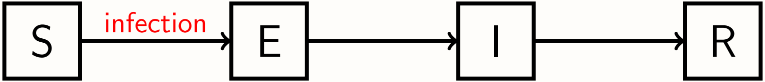

  - **Transmissions** require interaction with another class.
    - Susceptibles are infected by Infectives.
  - **Transitions** happen without any interaction.
    - Incubation of Latent individuals and removal of Infectious individuals happen spontaneously.

# Processes – Transitions



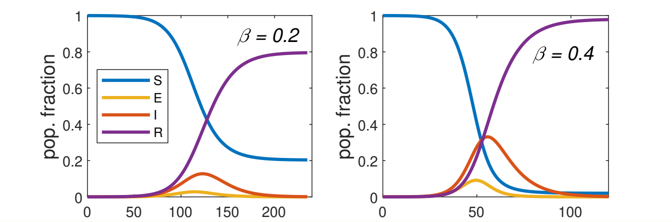

- Transition rates are (assumed to be) proportional to the **leaving** class
  - incubation rate $= \eta E$
  - removal rate $= \gamma I$
- Transition times are then exponentially distributed.
- **Rate constants are reciprocals of average time in class.**
  - Average removal time 10 days $\rightarrow \gamma = 0.1$

# Processes – Transmissions




- Transmission rates are proportional to the **leaving** class size
  - infection rate = force of infection $* S = \lambda S$
- The force of infection is proportional to the sum of the **transmitting** classes (just I for SEIR)
  - force of infection $= \beta I$
- $\beta$ is the product of the encounter rate and transmission probability, neither easily measured.

# How Does $\beta$ Affect Outcomes?




- It changes the peak $I$ value.
- It changes the final $S$ value.

# SEIR Final Size Relation

- We can relate $S$ to $R$, independent of time.

$$\frac{dR}{dS} = \frac{R'}{S'} = \frac{\gamma I}{-\beta SI} = -\frac{1}{\mathcal{R}_0}\frac{1}{S}.$$

1. Integrate from time 0 to infinity and nondimensionalize:
   - Let $s = S/N$, $r = R/N$, $s_0 = S(0)/N$, $r_0 = R(0)/N$.

$$\ln \frac{s_0}{s_\infty} = \mathcal{R}_0(r_\infty - r_0). \tag{1}$$

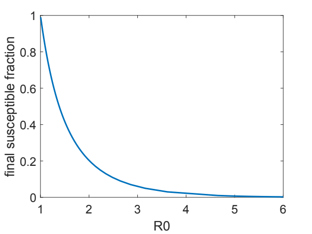
2. We can show that $e_\infty = i_\infty = 0$, so $s_\infty + r_\infty = 1$.

$$\ln \frac{s_0}{s_\infty} = \mathcal{R}_0(1 - r_0 - s_\infty). \tag{2}$$

# SEIR Model – Final size relation

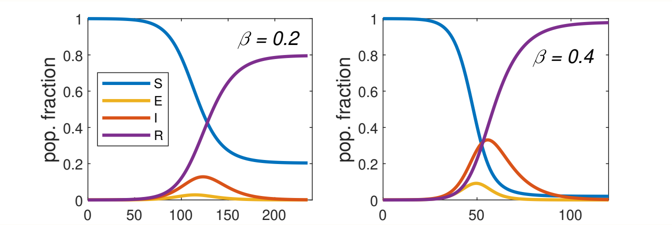
- Assume nearly everyone is susceptible, so $s_0 \approx 1$ and $r_0 = 0$.

$$\ln \frac{1}{s_\infty} = \mathcal{R}_0(1 - s_\infty). \tag{3}$$




- This works great if $\mathcal{R}_0 < 4$ and the epidemic ended without intervention. Then $\beta = \gamma \mathcal{R}_0$.

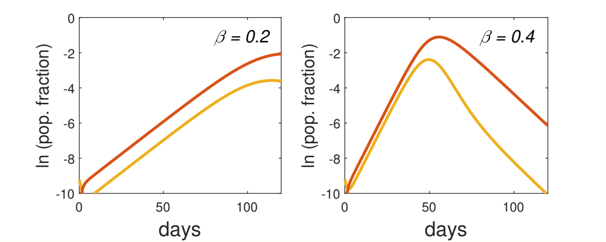
# How Does $\beta$ Affect Outcomes?




- It changes the peak $I$ value.
- It changes the final $S$ value.
- **Maybe it does something we can't see in these graphs.**

# How Else Does $\beta$ Affect Outcomes?

- Plot the logarithms of the infected populations $E$ and $I$.




- There is an extended period of exponential growth.

$$\ln I = \ln I_0 + \lambda t, \quad \ln E = \ln I + \ln \rho$$

for some $I_0$, $\rho$.

$$I = I_0 e^{\lambda t}, \quad E = \rho I.$$

# SEIR epidemic model – exponential phase

$$E' = \beta S_0 I - \eta E.$$
$$I' = \eta E - \gamma I.$$

With

$$E = \rho I, \quad I' = \lambda I, \quad E' = \rho\lambda I,$$

we get

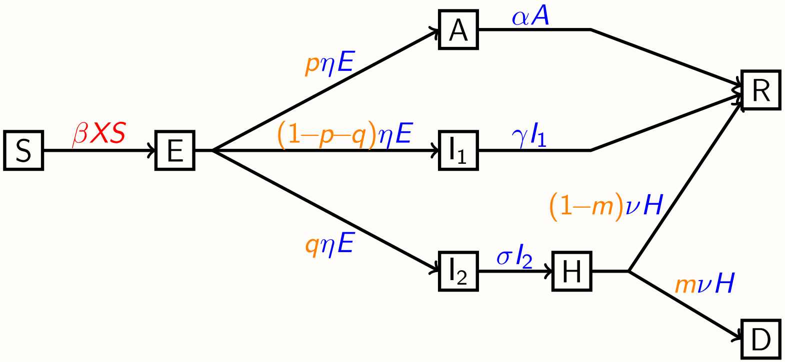
$$\beta = \frac{(\lambda+\eta)(\lambda+\gamma)}{S_0\eta}.$$

The growth rate $\lambda$ comes from the doubling time of class I:

$$\lambda = \frac{\ln 2}{t_d}.$$

So the doubling time gives us $\beta$.

# SEAIHRD Epidemic Model – March 2020 Scenario




transition rates, probabilities, transmission rates

# SEAIHRD Transmission Details (March 2020)


$$X = f_c c_i I + \delta[(1 - c_i)I + f_a A] \quad (4)$$

- $c_i$ is the fraction of confirmed cases for symptomatic infectives.[1]
- $f_c$ and $f_a$ are the infectivities of confirmed and asymptomatic cases, relative to an unconfirmed symptomatic infective.
- $\delta$ is a ‘contact factor’ that incorporates physical distancing and mask use for unconfirmed cases.

[1] There was no testing of asymptomatic people in spring 2020.

# Transition Rate and Infectivity Parameters

- $\eta$, $\alpha$, $\gamma$, $\sigma$, $\nu$ are the reciprocals of the mean durations for classes E, A, $I_1$, $I_2$, and $H$.
- Best estimates from data for these times are
  - 5 days for incubation
  - 8 days for asymptomatic infectiousness
  - 10 days for symptomatic infectiousness
  - 6 days for transition to hospitalization
  - 8 days for hospitalization
- Best estimate for infectivity of asymptomatics is $f_a = 0.75$.

# Probability Parameters

- An antibody study of a random sample of people estimated about 40% of asymptomatic cases, so $p = 0.4$.
- About 25% of hospitalized patients died, so $m = 0.25$.
- There is no direct data for the fraction of patients who required hospitalization.
  - One study estimated that roughly 1 of 11 total cases was confirmed by testing (counting both asymptomatic and untested symptomatics), with 12% of confirmed cases leading to hospitalization.
  - From this data, we can estimate that about 1.8% of symptomatic cases required hospitalization, so $q = 0.018$.
- The combination $q = 0.018$ and $m = 0.25$ results in a fatality rate of just under 0.5%, which would be 1.6M Americans, assuming everyone gets infected and there is no treatment.

# $\beta$ and Initial Conditions (and $\mathcal{R}_0$)

Assume exponential phase at start with hospitalization fraction $H_0$.

$$\frac{dE}{dt} = \beta S_0 X S - \eta E, \quad E(0) = e_0 H_0; \tag{5}$$

$$\frac{dA}{dt} = p\eta E - \alpha A, \quad A(0) = a_0 H_0; \tag{6}$$

$$\frac{dI_1}{dt} = (1 - p - q)\eta E - \gamma I_1, \quad I_1(0) = i_{10} H_0; \tag{7}$$

$$\frac{dI_2}{dt} = q\eta E - \sigma I_2, \quad I_2(0) = i_{20} H_0; \tag{8}$$

$$\frac{dH}{dt} = \sigma I_2 - \nu H, \quad H(0) = H_0; \tag{9}$$

$$X = I + f_a A \tag{10}$$

## $\beta$ and Initial Conditions (and $\mathcal{R}_0$)

- Unknowns $\beta$, $e_0$, $a_0$, $i_{10}$, $i_{20}$ can be given in terms of $\lambda$ and the other parameters.
- A history of estimates for $\mathcal{R}_0$:
  - December 2019: Published value of $\mathcal{R}_0 = 2.6$ based on statistical analysis of data from China.
  - January 2020 through March 2020: Numerous published values, most between 2.5 and 3.5, but one over 5.
  - April 8, 2020: Value $\mathcal{R}_0 = 5.0$ published on UNL COVID modeling web page and used in teaching modules for Math 203 and elsewhere. Based on hospitalization data from New York City.
  - July 2020: 'Definitive' result $\mathcal{R}_0 = 5.7$ published by Los Alamos research group (Sanche et al — similar method to mine, but better data).

# Public Health Parameters

- The public health parameters are difficult to estimate and highly variable.
- The testing fraction for symptomatic patients $c_i$ was probably in the range 0.1 to 0.6. This parameter is a good choice for a parameter study showing the impact of testing.
- The infectivity of isolation can only be guessed. I use $f_c = 0.1$.
- The contact factor $\delta$ is another good choice for a parameter study. Cell phone data suggests values from 0.2 to 0.6.

# Some Conclusions

- Looking at data in different ways can yield insights that you might have missed.
- 'Nobody believes a model except the person who created it; everyone believes data except the person who collected it.'[2]
  - We should be equally sceptical of models and data.
- It is often easier to measure a parameter by its effect than by direct measurement.
- If you get bad results from a good model, then at least one parameter value is wrong.

[2] Paraphrase of a quotation from Albert Einstein.