

# Mathematical Model FAQ

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## 1. What is a mathematical model?

A *mathematical model* is a collection of one or more dependent variables together with enough mathematical equations or rules to prescribe the values of those variables in terms of given input quantities. Models are designed with the goal of being able to predict what happens in a real-world setting.

## 2. Where do models come from?

If we collect data and plot a graph, we can try to find a known function whose graph resembles the data. A model chosen this way is an *empirical* model.

If we use assumptions based on scientific principles, we can make a *mechanistic* model. The linear motion model  $d = rt$  is a mathematical rendering of the physical assumption that a particular object is moving at a constant speed  $r$  and achieves distance  $d$  in time  $t$ . Similarly, the simplest falling body model,  $v = gt$ , follows from the physical assumption that the rate of change of the downward velocity of a dropped object is a constant ( $g$ ). This model includes the additional assumption that all forces other than gravity can be neglected.

## 3. What is the relationship between models and reality?

Models do not correspond to reality. They correspond to a caricature of reality, just as a political cartoon uses a caricature of a public figure rather than a photograph. Figure 1 illustrates this using the idea of a *conceptual model* for the caricature of reality. The conceptual model is not always fully stated. (For example, the fact that other forces are neglected is not always included in the description of the model  $v = gt$ .)

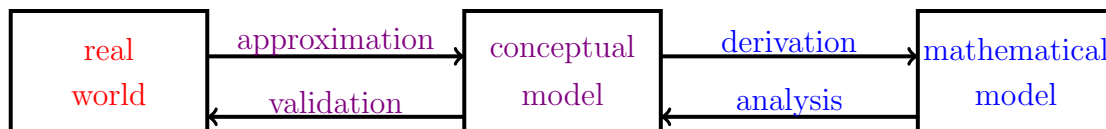


Figure 1: Relationship of a mathematical model to the real world.

4. What is a ‘parameter’?

A *parameter* is a quantity in a mathematical model that is fixed for a particular instance of a model but can vary from one instance to another. The gravitational constant  $g$  in the falling body model  $v = gt$  is a parameter that allows us to use the same model for the Earth and the moon, simply by using a different value.

5. How do we know if a model is right or wrong?

This is like asking how we know if a thought is green. ‘Right’ and ‘wrong’ are inappropriate terms for mathematical models. They aren’t supposed to match reality, so there is no standard by which they can be judged right or wrong. Mathematical models are ‘good’ in a given context if they give us useful information for that context and ‘bad’ if they don’t.

6. Why do we use models if they can’t be ‘correct’?

It is always best in science to learn things by experiment. Models are the only available alternative in cases where we can’t do the experiment, like if we want to know how the number of people who die from the delta variant of COVID-19 depends on the fraction of the population who are fully vaccinated. A good COVID-19 model can make excellent qualitative predictions for questions like this.

7. What does it mean to say that model outputs are “a function of the parameters”?

A *function* is a term used in mathematics to mean that output quantities have values that are completely determined by the input quantities. In a disease model, the total number of deaths (a model output) is a function of two key parameters, one representing the infectiousness of the disease and the other representing the fraction of infected people who die.